Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

Non-Parametric Rank Statistics for Spectral Power and Coherence

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Dublin, Ireland

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Trinity College Dublin, the University of Dublin

Outline

0. **Motivation (Application Domain)**
1. **Introduction**
2. **Original Formulation for Spectra and Coherence**
3. **Non-Parametric Estimation of Spectra and Coherence**
4. **No-Parametric Rank Statistics for Spectral Power**
    1. 1-Sample Power
    2. 2-Sample Power
5. **No-Parametric Rank Statistics for Coherence**
    1. 1-Sample Coherence
    2. 2-Sample Coherence (Phase, Magnitude, Both)
6. **Numerical Examples**
    1. Simulated Data
    2. Experimental Data
7. **Discussion**
8. **Conclusions and Recommendations**

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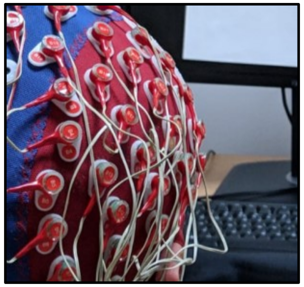
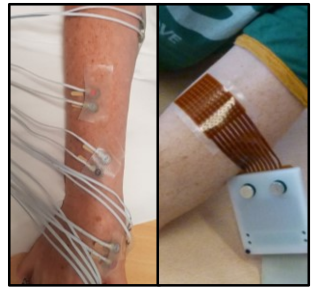
Motivation

Targeting and Interrogating Networks by Novel Neuro-electric Biomarkers

Techniques



EEG (with source Analysis)



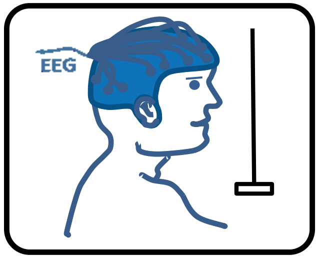
EMG (multi-channel/HD)

TMS (+ Threshold Tracking)

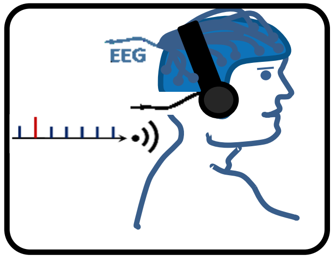
Motivation

Targeting and Interrogating Networks by Novel Neuro-electric Biomarkers

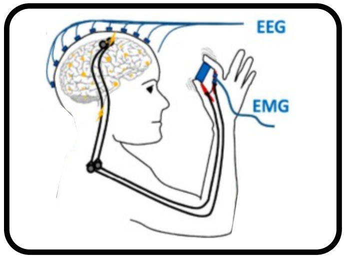
Paradigms



Resting State



Cognitive Tasks

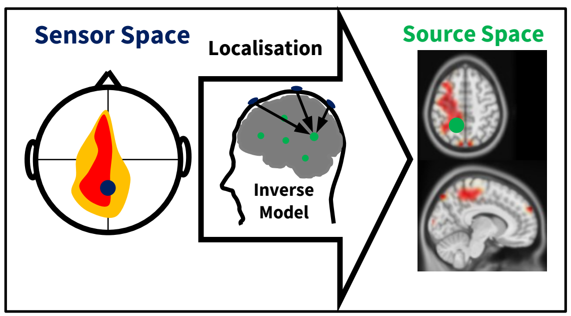


Motor Tasks

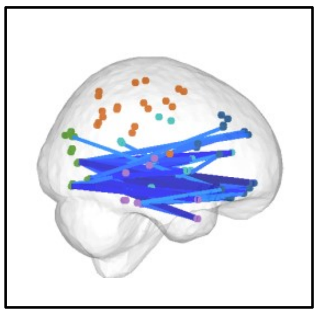
Motivation

Targeting and Interrogating Networks by Novel Neuro-electric Biomarkers

Analysis



Brain **Source Reconstruction**

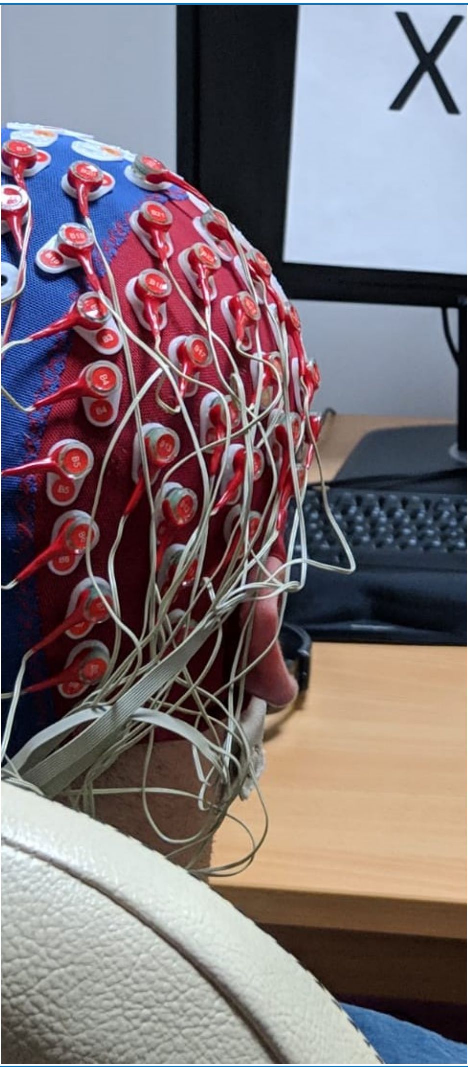


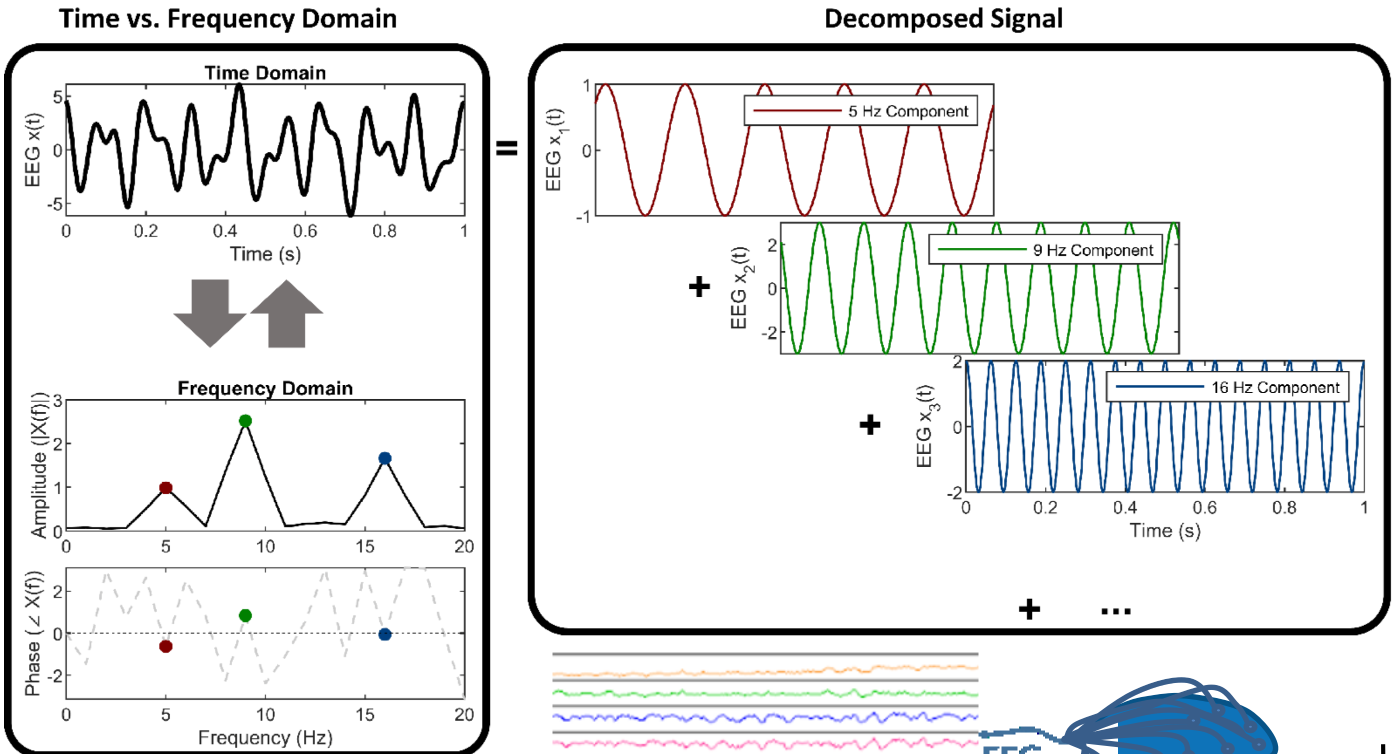
Connectivity Analysis

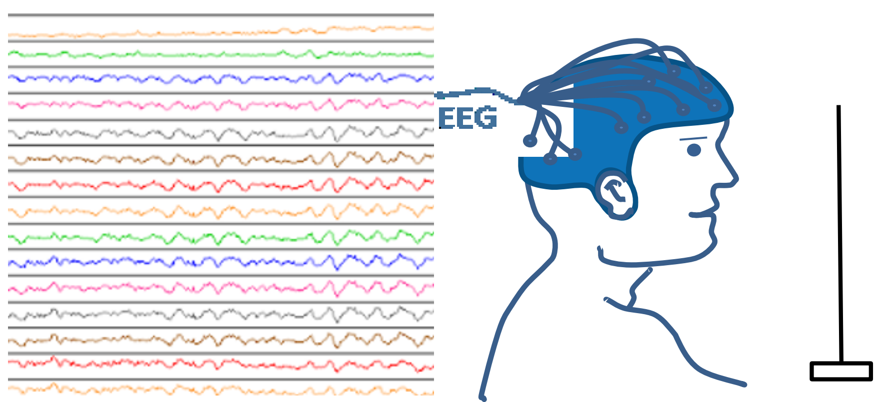
(McMackin *et al.*, 2019, J Neurol Neuros Psych)

Resting-State EEG Networks

Resting State: Continuous EEG Recordings





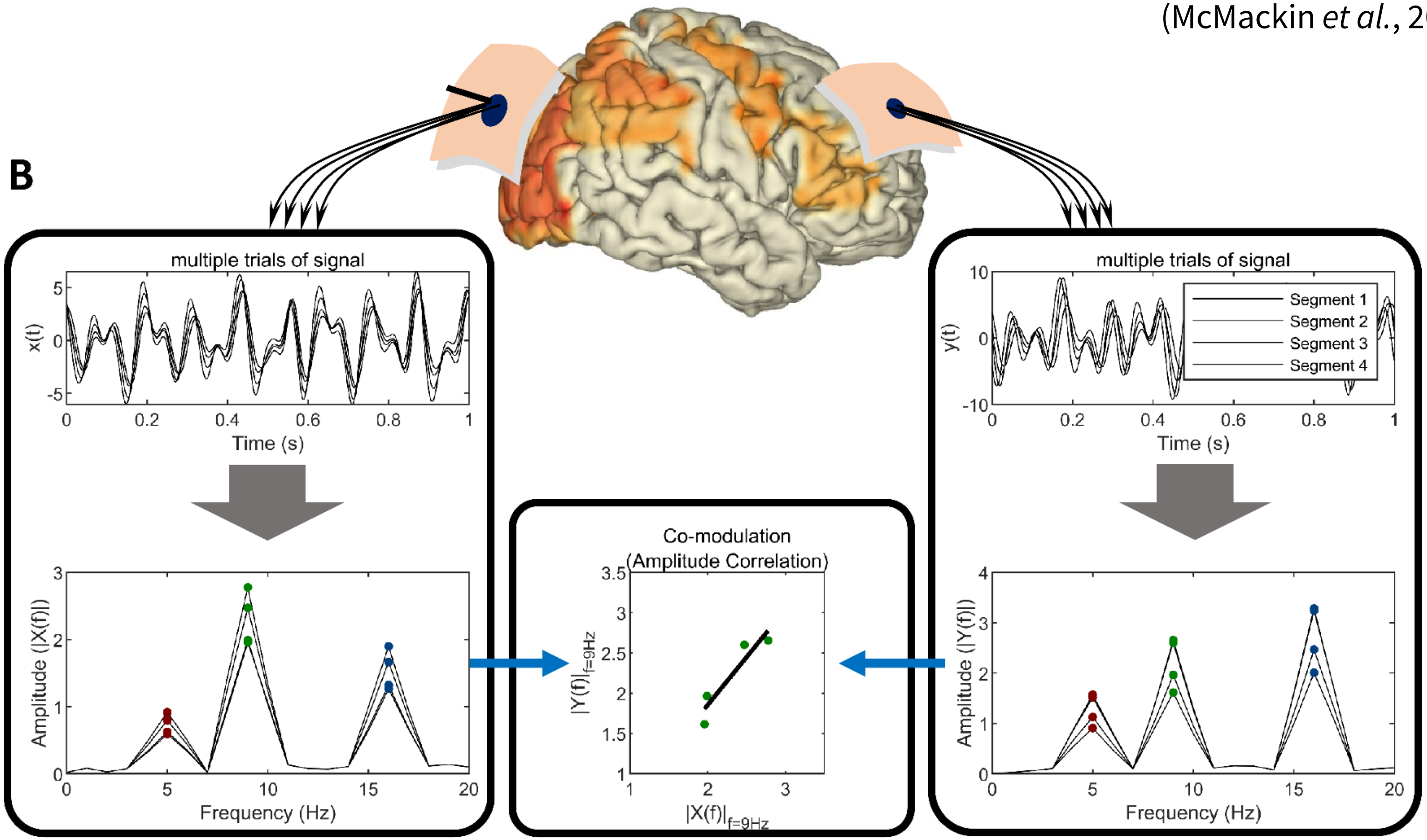


(McMackin *et al.*, 2019, JNNP)

Motivation: Resting-State EEG Networks

Spectral Power and Connectivity (Co-Modulation and/or Synchrony)

(McMackin *et al.*, 2019, JNNP)

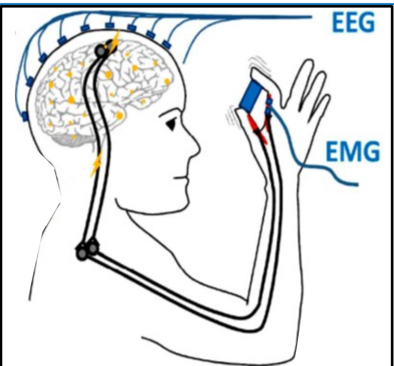


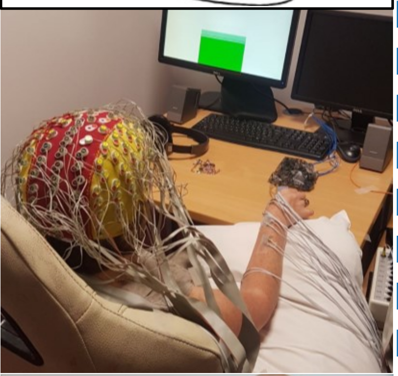
Signal x from Region 1 **Connectivity bw. Regions 1 & 2** **Signal y from Region 2**

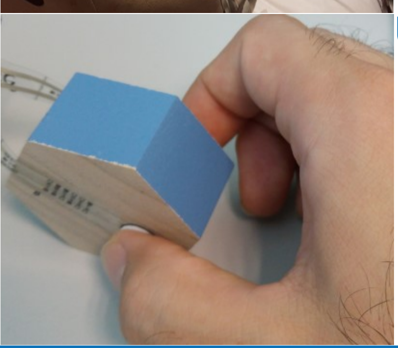
Frequency-Specific (Spectral) Analysis of Power and Connectivity/Synchrony

Motivation: Motor Networks

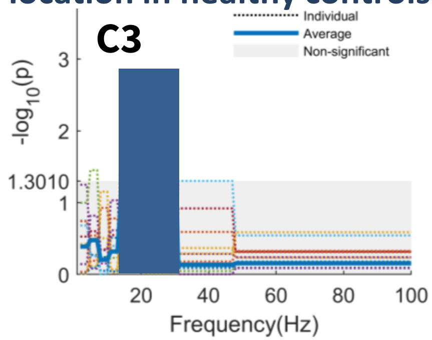
Motor Tasks: EEG-EMG Coherence

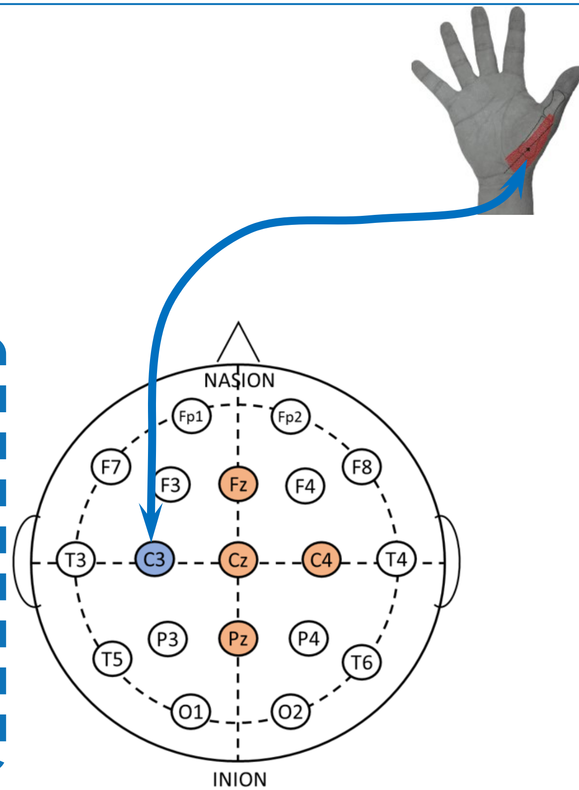






Typical Beta band frequency & location in healthy controls

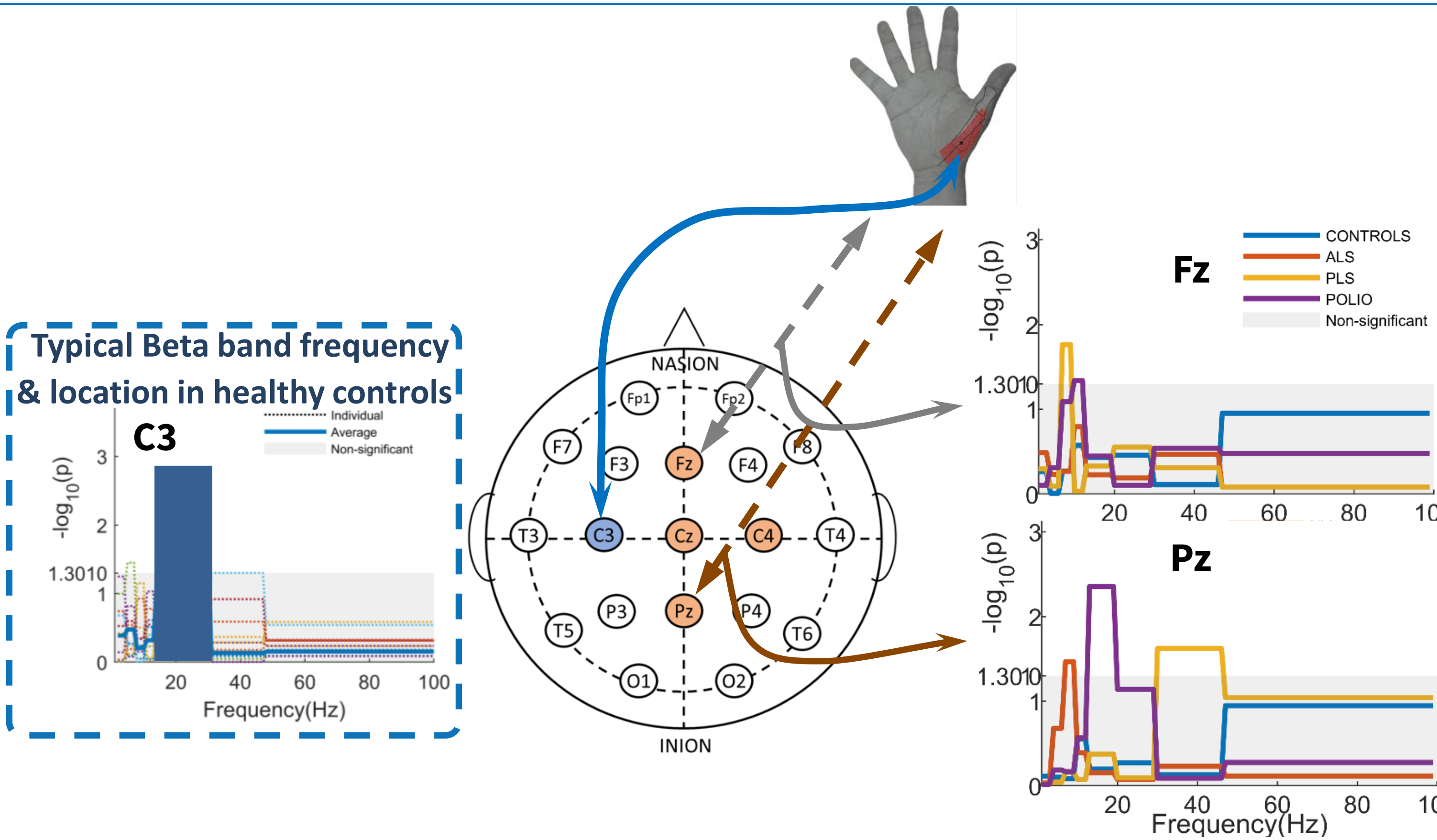




Coffey, et al., 2021, Clin. Neuroph.

Motivation: Motor Networks

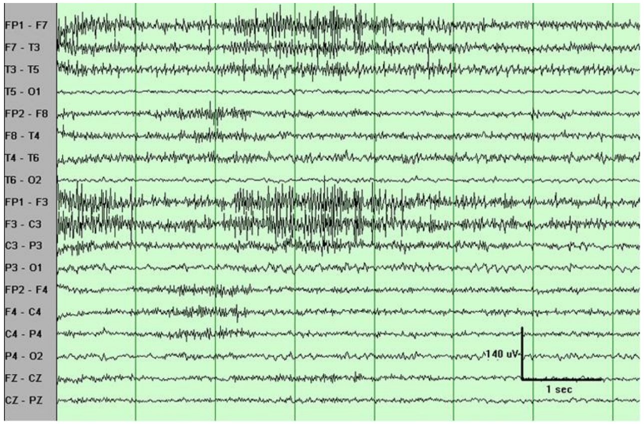
Increase Abnormal EEG-EMG Coherence in MND/ALS



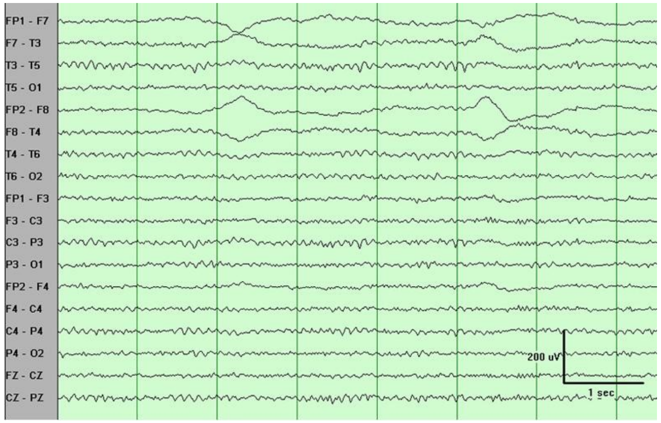
Coffey, et al., 2021, Clin. Neuroph.

Non-motor locations & abnormal frequencies in PLS, POL, ALS show compensatory activity in other cortico-spinal networks

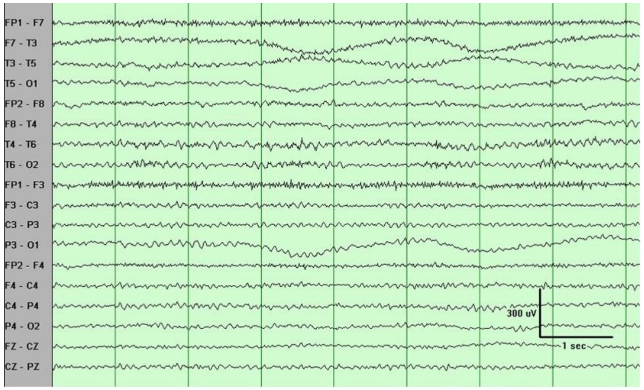
Motivation Examples of EEG vs Artefacts



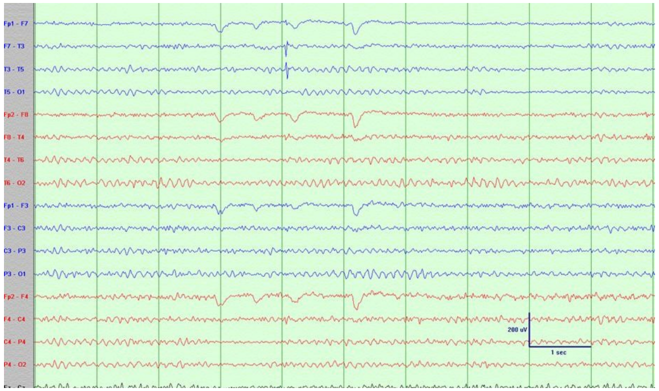
EMG Artefacts



Eye Movement Artefacts



Sweet Artefacts

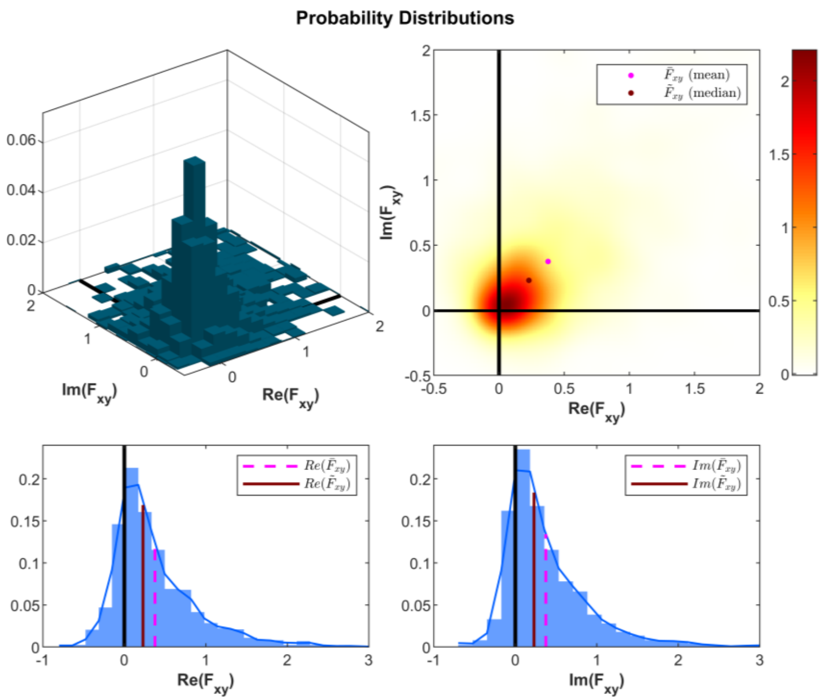


Electrode Artefacts

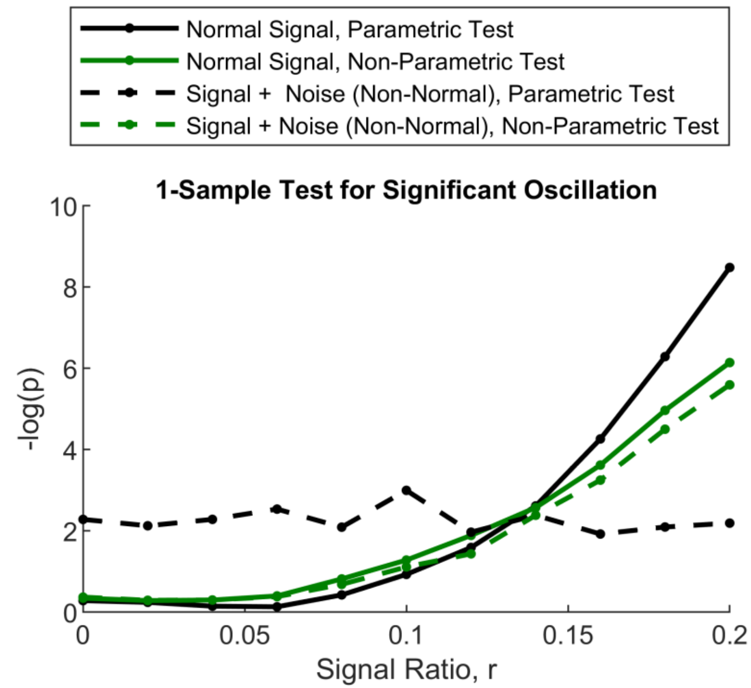
http://emedicine.medscape.com/article/1140247-overview

Overview

Non-Parametric Rank Statistics for Spectral Power and Coherence



Robustness of Non-Parametric Tests for Simulated Spectral Power



Nasseroleslami et al., BioRxiv

Non-Parametric-based Estimates of Power and Coherence are Robust against Artefacts

Dukic *et al.*, 2017, IEEE EMBC 2017

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Introduction

- Parametric Estimation of Spectral Power and Coherence in (Neural) Time Series Analysis is documented (Brillinger, 2001; Halliday and Rosenberg, 1999)
- The statistical inference of spectral power and coherence of neural signals remains a practical challenge.
  - **Non-normal Distribution**
  - **Artefactual Components**
  - **Bias**
  - **Complex statistical distributions.**

Introduction

- To parallel, Sign rank, Mann-Whitney tests, we need non-parametric methods.

- Non-parametric methods (e.g. based on median) of spectra afford robust estimation (Dukic et al. 2017).

Estimation of Coherence using the Median is Robust against EEG Artefacts

Stefan Dukic, Parameswaran Mahadeva Iyer, Kieran Mohr, Orla Hardiman, Edmund C. Lalor, Bahman Nasseroleslami

- However, the statistical inference based on these non-parametric estimates remain to be formulated and tested.

Aim and Objective:

- **Aim:** To provide **non-parametric rank tests** for 1- and 2-sample statistical testing of **spectral power** and **coherence**

- To demonstrate and verify the non-parametric tests using simulated and real neural signals in different conditions, and to assess their robustness in presence of artefactual components.

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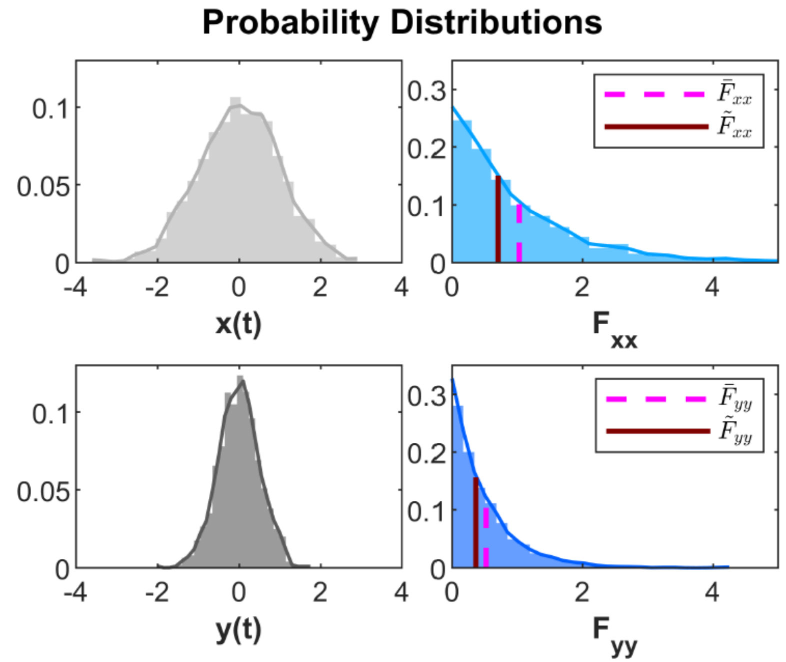
Spectral Power and Coherence

Spectral Power

- Consider x(t) and y(t) to be time domain signals

$$
\begin{aligned}
F_{xx}(f) &= \mathcal{E}\{X_i(f)X_i(f)^*\} \\
F_{yy}(f) &= \mathcal{E}\{Y_i(f)Y_i(f)^*\} \\
F_{xy}(f) &= \mathcal{E}\{X_i(f)Y_i(f)^*\}
\end{aligned}
$$

$$
\begin{aligned}
\bar{F}_{xx}(f) &= \frac{1}{L}\sum_{i=1}^{L} X_i(f)X_i(f)^* \\
\bar{F}_{yy}(f) &= \frac{1}{L}\sum_{i=1}^{L} Y_i(f)Y_i(f)^* \\
\bar{F}_{xy}(f) &= \frac{1}{L}\sum_{i=1}^{L} X_i(f)Y_i(f)^*
\end{aligned}
$$



- Statistics:

$$var\{F_{xx}(f)\} \approx (F_{xx}(f))^2/L$$

Spectral Power and Coherence

Coherence

- Coherency function C and coherence $|C_{xy}(f)|^2$

$$C_{xy}(f) = \frac{F_{xy}(f)}{\sqrt{F_{xx}(f)F_{yy}(f)}}$$

- Statistics:
- Coherence has a hypergeometric (sampling) distribution. Under null hypothesis (0 coherence), $tanh^{-1}(.)$ provides an approximate transformation to normal.

$$p = (1 - |C_{xy}(f)|^2)^{(L-1)}$$

Brillinger 2004; Halliday et al 1995; Priestly 1982

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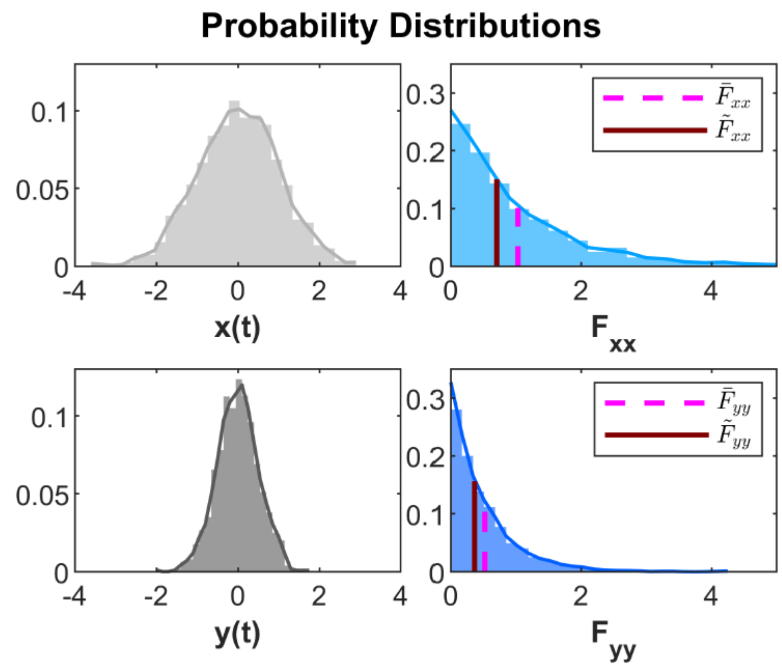
F_{xy}(f) &= \mathcal{E}\{X_i(f)Y_i(f)^*\}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{F}_{xx}(f) &= Median_i[\{X_i(f)X_i(f)^*\}] \\
\tilde{F}_{yy}(f) &= Median_i[\{Y_i(f)Y_i(f)^*\}] \\
\tilde{F}_{xy}(f) &= Median_i[\{X_i(f)Y_i(f)^*\}]
\end{aligned}
$$

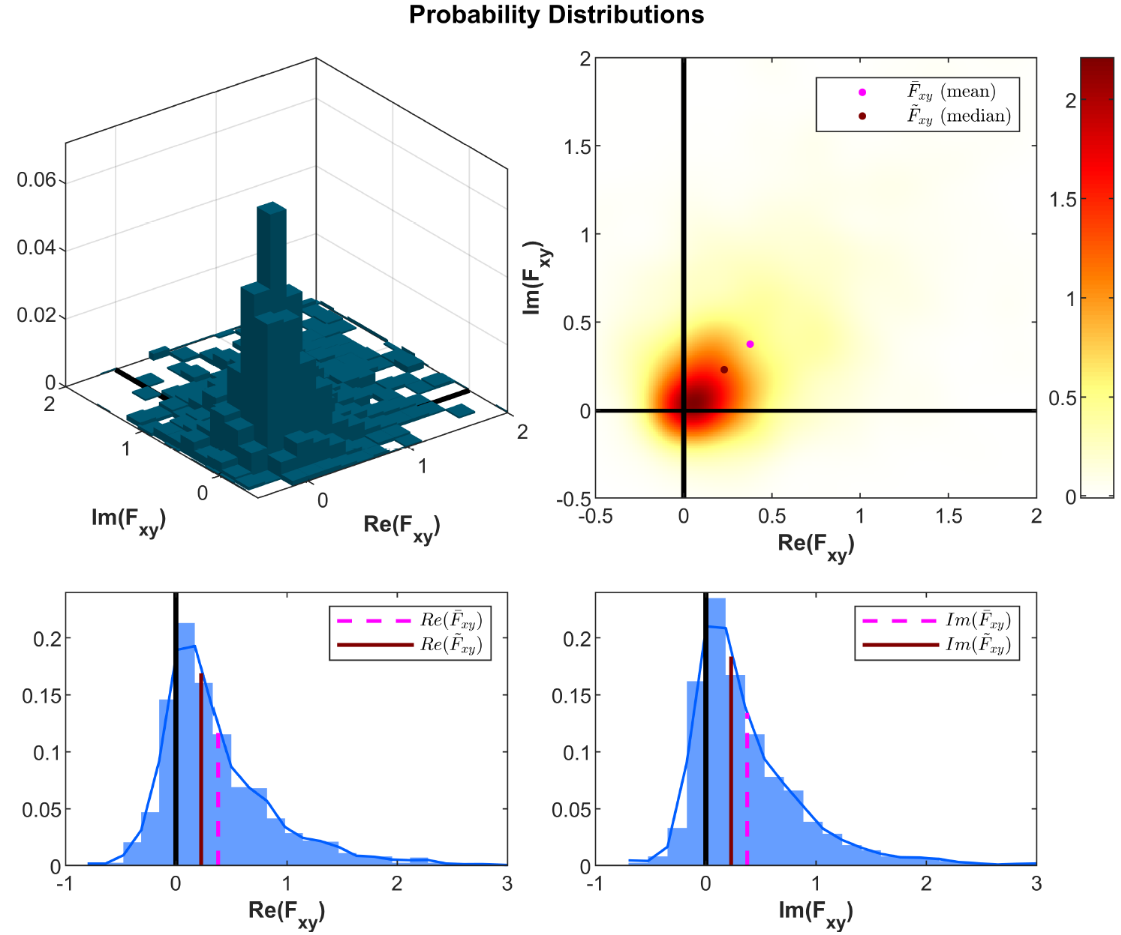


- **Spatial Median:**

$$\tilde{F}_{xy}(f) = \arg\min_{\Theta}\left(\sum_{i=1}^{L} \|X_i(f)Y_i(f)^* - \Theta\|\right)$$

Non-Parametric Spectral Power and Coherence

Coherence



Statistical Distribution of the Raw Cross-Spectra in 2D and Marginal Plots and the Estimation of Cross-Spectrum using the Mean and Median (~ Fxy).

Non-Parametric Spectral Power and Coherence

Coherence

- Consider x(t) and y(t) to be time domain signals

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F_{xx}(f) &= \mathcal{E}\{X_i(f)X_i(f)^*\} \\
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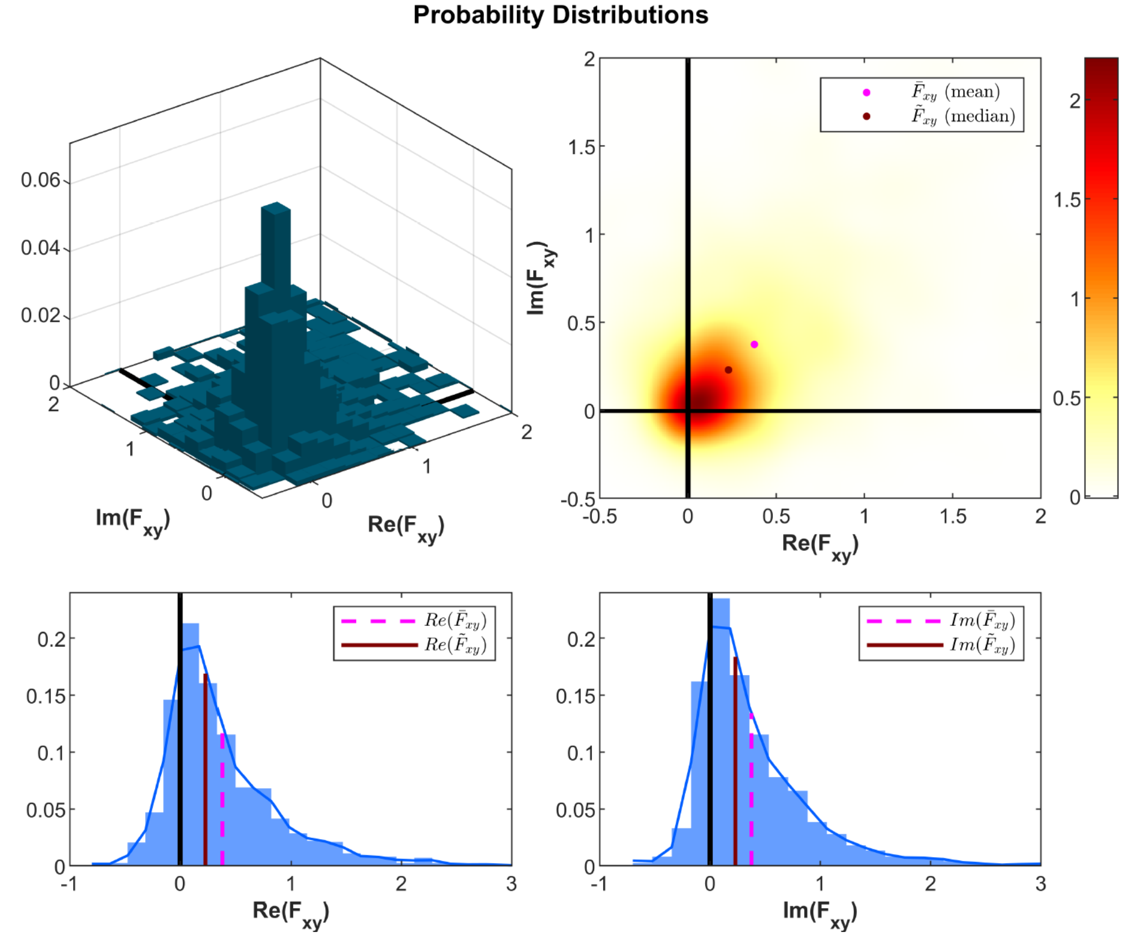
- **Spatial Median:**

$$\tilde{F}_{xy}(f) = \arg\min_{\Theta}\left(\sum_{i=1}^{L} \|X_i(f)Y_i(f)^* - \Theta\|\right)$$

$$\tilde{C}_{xy}(f) = \frac{\tilde{F}_{xy}(f)}{\sqrt{\tilde{F}_{xx}(f)\tilde{F}_{yy}(f)}}$$

Non-Parametric Spectral Power and Coherence

Coherence



Statistical Distribution of the Raw Cross-Spectra in 2D and Marginal Plots and the Estimation of Cross-Spectrum using the Mean and Median.

Outline

Non-Parametric Rank Statistics for Spectral Power

Main change in the way we handle time series statistics:

Exploiting Individual and All Data Points Regardless of Their Measures and Their (Sampling) Distributions

Non-Parametric Rank Statistics for Spectral Power

Confidence Intervals from Raw Spectra:

$$[iCDF_{\{|X_i(f)|^2\}}(\alpha/2), iCDF_{\{|X_i(f)|^2\}}(1-(\alpha/2))]$$

Non-Parametric Rank Statistics for Spectral Power

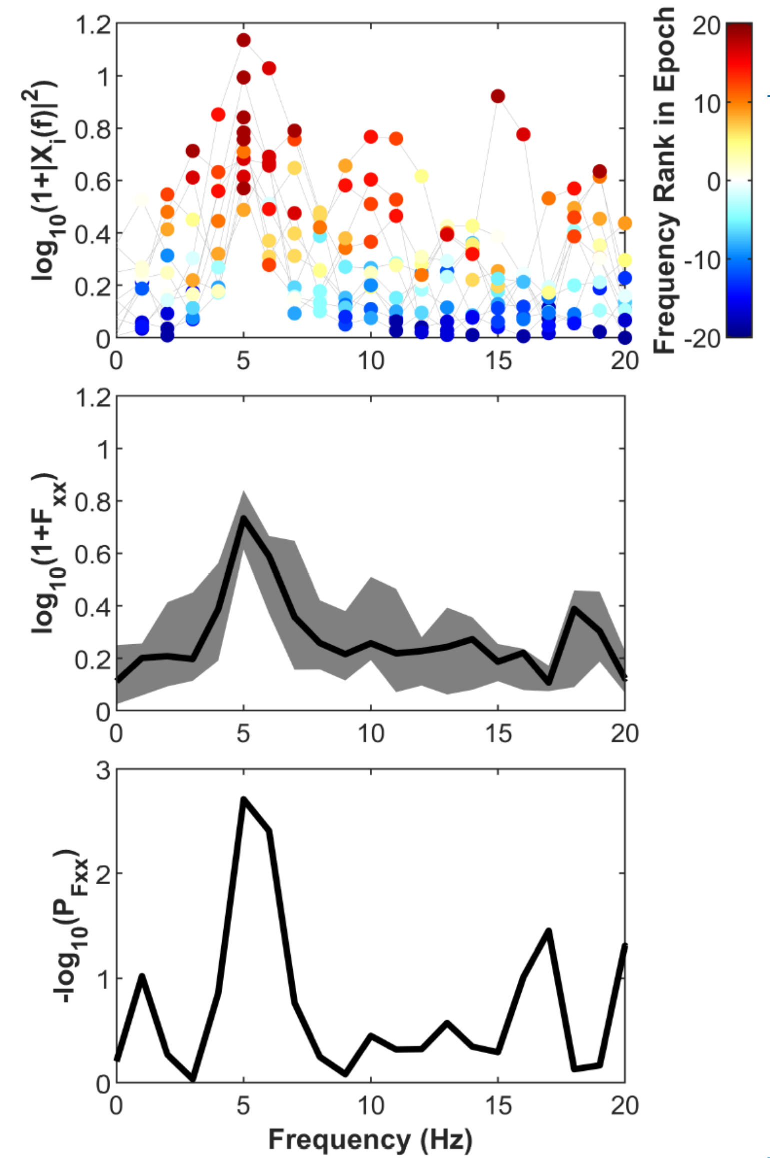
1 Sample

Typical Hypothesis: Presence of significant (decrease/increase) in specific frequencies (compared to white noise)

One-Sample Significant Power: Wilcoxon's Signed Rank test on the centred rank values

2 Sample

Two-Sample Significant Power Difference: Traditional Mann-Whitney U test.



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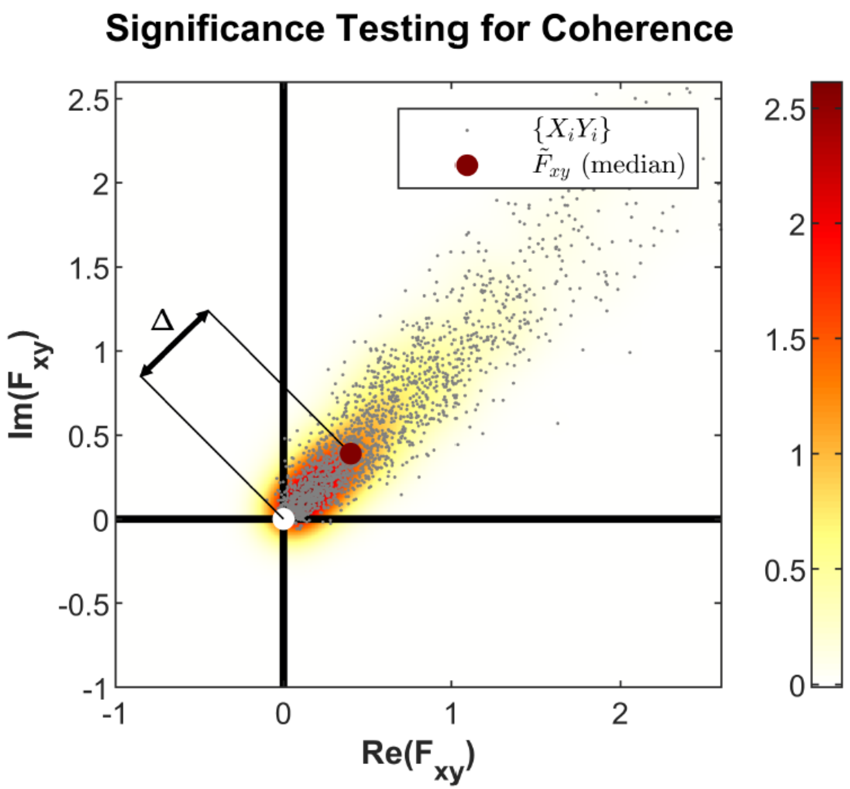
Non-Parametric Rank Statistics for Coherence

1 Sample

1-Sample Location Problem

Test Selection Criterion:

Affine Invariant



One-Sample Significant Coherence: One-Sample Spatial (signed) Ranks Test (Hannu, Oja & Randles, 2004; Hannu, Oja, 2010; Nordhausen & Oja, 2011).

Non-Parametric Rank Statistics for Coherence

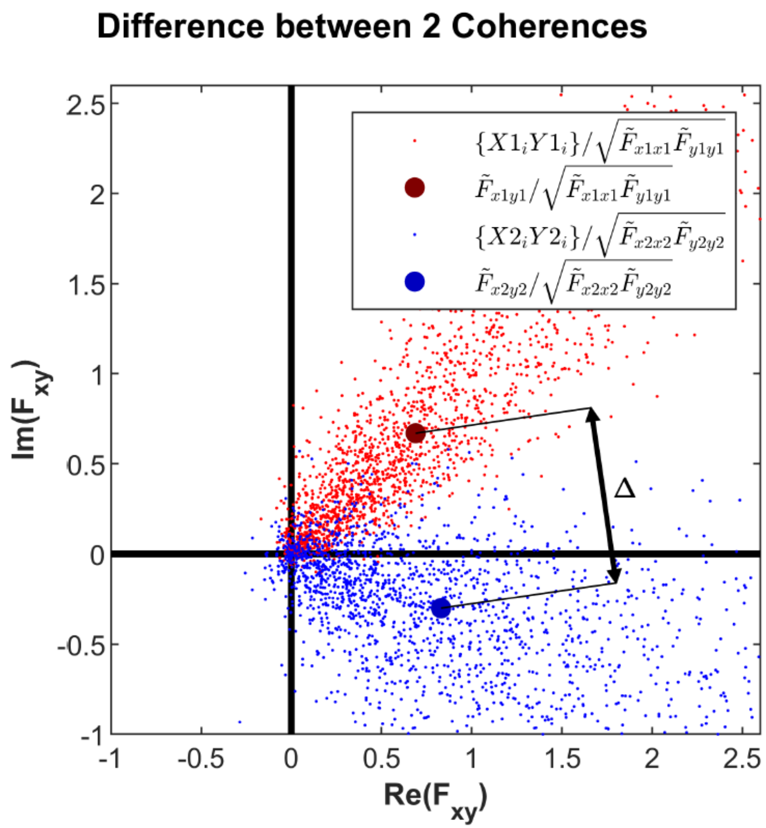
2 Sample

2-Sample Location Problem

Test Selection Criterion:

Affine Invariant

- Reflects Phase or Magnitude Difference



Two-Sample Significant Coherence Difference (Magnitude and Phase): Two-Sample Spatial Ranks Test (Hannu Oja & Randles, 2004; Hannu Oja, 2010; Nordhausen & Oja, 2011).

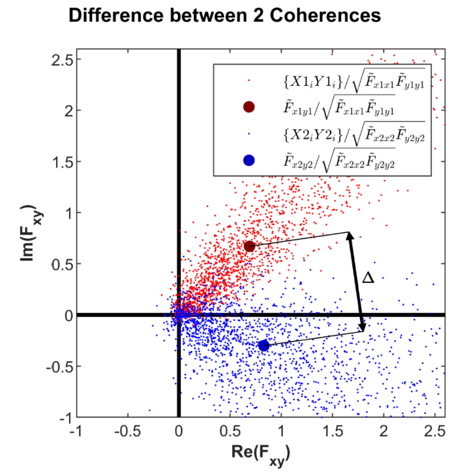
Non-Parametric Rank Statistics for Coherence

2 Sample: Separate Testing for Phase or Magnitude

- **Two-Sample Significant Difference in Coherence Magnitude:** Statistical trick by subtraction of z-scores, similar to Stouffer's method.

$$
\begin{aligned}
z_1 &= iCDF_{\mathcal{N}(0,1)}(p_1) \\
z_2 &= iCDF_{\mathcal{N}(0,1)}(p_2) \\
P_{diff,L} &= CDF_{\mathcal{N}(0,1)}(\frac{z_2 - z_1}{\sqrt{2}}) \\
P_{diff,R} &= 1 - P_{diff,L} \\
P_{diff} &= 2.min(P_{diff,L}, P_{diff,R})
\end{aligned}
$$

- **Two-Sample Significant Difference in Coherence Phase:** Mann-Whitney U Test between the global-mean-subtracted phase values. (or other circular statistical tests)



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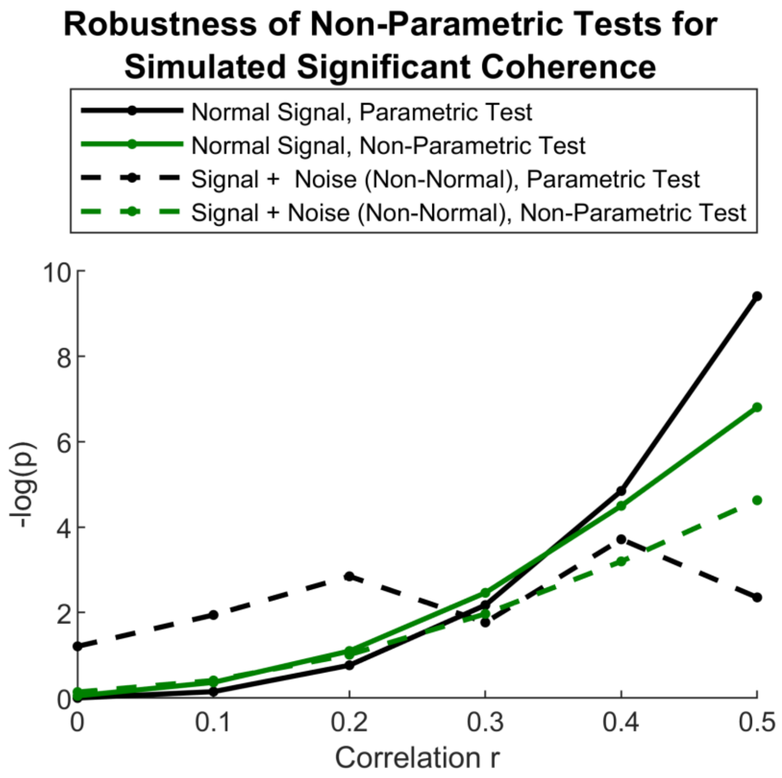
Numerical Examples

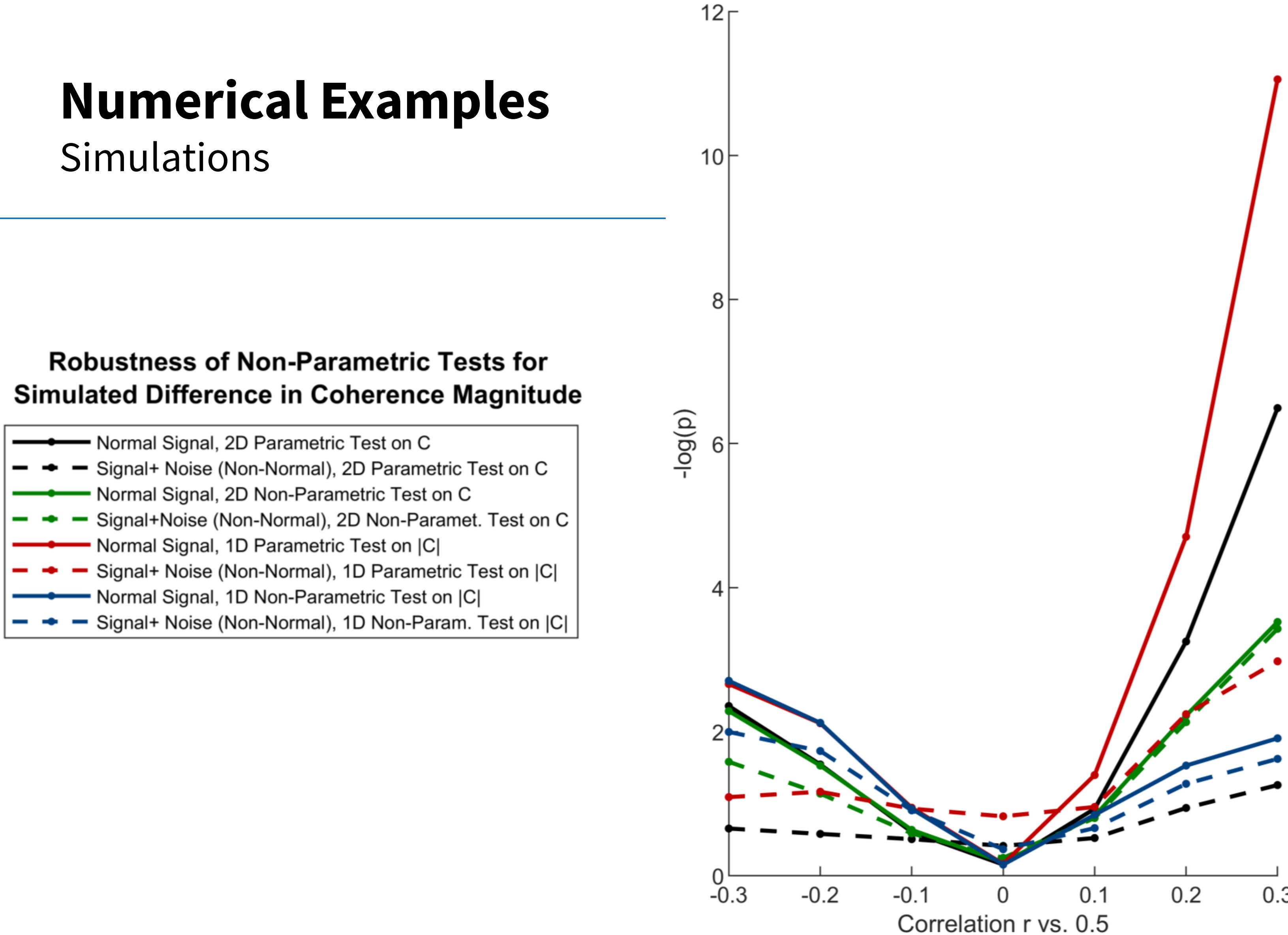
Simulations



The 1-Sample Spatial Signed Rank, is Robust Against Artefacts.

Numerical Examples

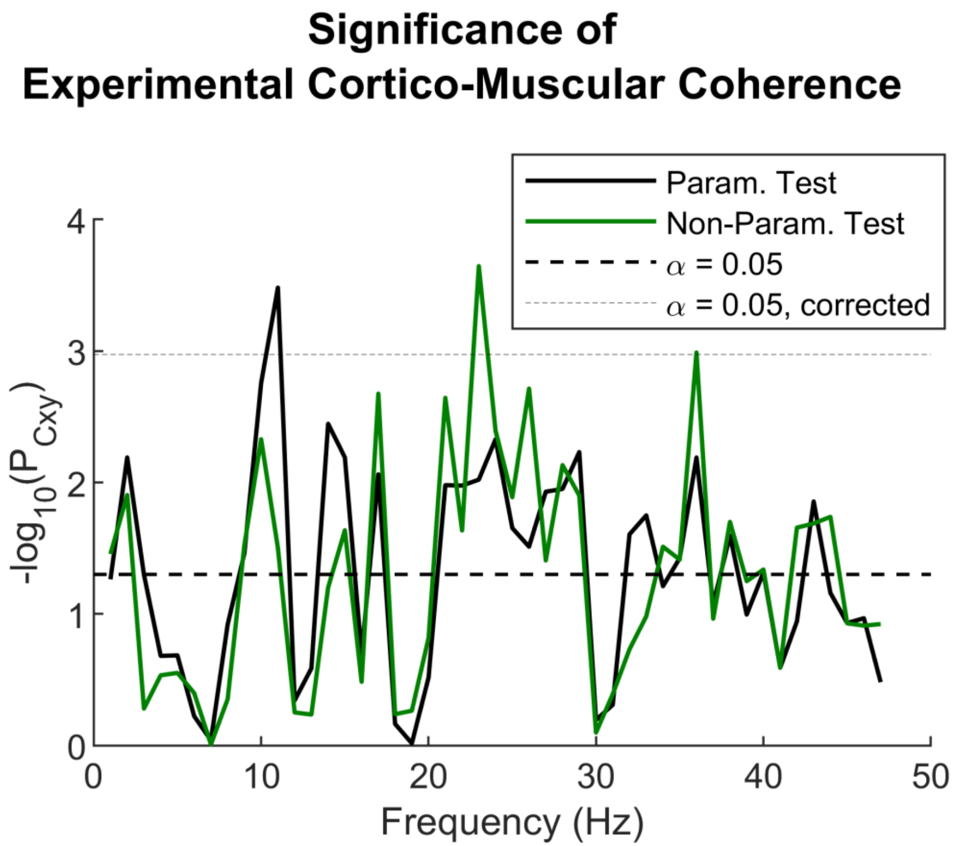
Simulations



The 2-Sample Statistical Tests based on SpatialRank and Spatial SignedRank are Robust Against Artefacts

Numerical Examples

Real Data



Both Test Families Detect Significant Presence of Coherence Between EEG and EMG signals.

Figure: Difference Between Cortico-Muscular Coherence in Left/Right Hemispheres.

Outline

Discussion

- **Benefits:** Robustness, Distribution-free, Allowing Testing for Magnitude/Phase, No need for Bootstrapping.

- **Challenges:** Closed form solutions, lower statistical power and sensitivity

- **Opportunities:** Usable for Time-Frequency, and Partial spectral and Coherence, and other time series, can be tested at individual subject or group-level.

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Recommendations

Summary of Tests

- **One-Sample Significant Power:** Wilcoxon's Signed Rank test on the centred rank values [1].
- **Two-Sample Significant Power Difference:** Traditional Mann-Whitney U test.
- **One-Sample Significant Coherence:** One-Sample Spatial (signed) Ranks Test (Hannu, Oja & Randles, 2004; Hannu, Oja, 2010; Nordhausen & Oja, 2011).
- **Two-Sample Significant Coherence Difference (Magnitude and Phase):** Two-Sample Spatial Ranks Test (Hannu Oja & Randles, 2004; Hannu Oja, 2010; Nordhausen & Oja, 2011).
- **Two-Sample Significant Difference in Coherence Magnitude:** Statistical trick by subtraction of z-scores, similar to Stouffer's method [1].
- **Two-Sample Significant Difference in Coherence Phase:** Mann-Whitney U Test between the global-mean-subtracted phase values.

Conclusions

- The approach provides **a new framework for non-parametric statistical analysis of the neural signal spectra**.
- These methods are suited to **neuroscience & neural engineering applications**, given the attractive properties such as **minimal assumption** on distributions, **statistical robustness**, and the diverse testing scenarios afforded.

Reference

Nasseroleslami B, Dukic S, Bista S, Buxo T, Coffey A, McMackin R, Muthuraman M, Hardiman O, Lalor EC, Lowery MM. 2019. “Non-Parametric Rank Statistics for Spectral Power and Coherence”. *bioRxiv.:818906*. **doi:10.1101/818906**.

Thank you

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- Trinity College Institute of Neuroscience (TCIN), Neural Engineering Group.
- Clinical Teams is St. James's Hospital, Beaumont Hospital.
- Wellcome-HRB Clinical Research facility (CRF), at St. James's Hospital
- Participants and Patients and their families

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rmn Research Motor Neurone
mnd.ie (MRCG-2018-02)

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin
www.tcd.ie (FoHS, Neurology, Mathematics)

IRISH RESEARCH COUNCIL
An Chomhairle um Thaighde in Éirinn
research.ie
(**GOIPD/2015/213**, GOIPG/2017/1014, GOIPG/2019/748)

HRB Health Research Board
hrb.ie (MRCG-2018-02)

THE ROYAL SOCIETY
royalsociety.org (IES\R3\183107)

future neuro
futureneurocentre.ie

MERCK
merckgroup.com

Fondúireacht Eolaíochta Éireann sfi Dá bhfuil romhainn
sfi.ie

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Thank You